Nuclear Radiation Perturbation of a Semiconductor-Filled Microwave Cavity

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Abstract—**The perturbation of a semiconductor-filled microwave cavity by nuclear radiation particles is considered as a possible means of radiation counting. The TE_r mode equations for a spherical cavity are summarized, and the charge diffusion-recombination equation is solved for this spherical geometry. The case of a spherical cavity operating in the TE_{011} mode, with the charge ionization occurring at the center, is analyzed. Through the utilization of a semisteady-state approach, a normalized expression for the power change as a function of time is obtained. This quantitative description for the output pulse shape is applied to the case of high-purity CdS at a frequency of 46.5 Gc/s. These results are presented in an accompanying figure.**

Introduction

In recent years, considerable interest has been generated in the measurement of the microwave properties of semiconductors [1]–[3]. A great deal of progress has also been made toward microwave applications of semiconductor devices [4]–[6]. A relatively untested concept in the applications field is the utilization of the microwave properties of semiconductors for the detection and the counting of high-energy particle radiation.

The conventional techniques used in particle detecting and energy spectroscopy involve the use of dc mode counters [7]. Radiation particles impinge upon the surface of a semiconductor, either in the form of a simple block or in the form of a reverse biased *p-n* diode. The radiation-induced charge carriers are swept to electrodes by either external or internal fields, collected, and eventually counted. Failure to collect all the charge can depreciate the signal amplitude and the energy resolution of the system. An ac mode counter, on the other hand, relies upon the change in the complex dielectric properties of the semiconductor. Thus, it is only necessary that nonequilibrium charges exist; they need not be collected. This implies that ohmic contacts to the semiconductor are not required. Also, larger crystals could be used, so that beta spectroscopy might be possible.

Initial steps toward the development of ac mode counters have been taken by Borisov and Marinov [8]. Their experiment consisted of a CdS crystal placed in parallel with the tank circuit of a 1-Mc/s oscillator. The output of the oscillator was attenuated as the losses in the crystal were pulsed by radiation. Resolutions of approximately 0.5 MeV were obtained for 8.77 MeV alpha particles from thorium C^1. It would seem that better time resolution could be obtained at microwave frequencies since the pulse rise time would be limited only by the frequency used. This means that the perturbations due to single particles could be more readily measured. The decay time would be limited by the recombination process, so that materials having inherently short lifetimes might be desirable. The problems associated with "pile-up" would be the same as for dc mode counters, assuming the particle rate could ever become this high. This is another topic for analysis within itself and will not be dealt with here. The fast rise time also indicates that events occurring very shortly after ionization could be investigated, if desired.

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Cavity Perturbation

One technique which might be utilized in ac mode counting is the power loss resulting from the perturbation of a cavity containing the semiconductor sample. A spherical cavity would be useful in this case because of its inherently higher Q. Also, the spherical geometry readily allows calculation of the diffusion effect, which has spherical symmetry.

Prior to a discussion of the ac mode cavity techniques, the Transverse Electric (TE) to r mode equations of the cavity will be briefly summarized. The TE_r fields may be derived by substituting the mode function [9]

$$F = \mathbf{a}_r F_r = \mathbf{a}_r F_0 \hat{J}_p(kr) P_p^m(\cos\theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \tag{1}$$

into the expressions

$$E = -\nabla \times F$$
$$H = \frac{1}{j\omega\mu}(\nabla \times \nabla \times F). \tag{2}$$

In (1) and (2), $\mathbf{a}_r$ is a unit vector in the r direction, F_0 is an amplitude factor, $\hat{J}_p(kr)$ is the spherical Bessel function, k is the propagation factor, and $P_p^m(\cos\theta)$ is an associated Legendre function. To satisfy the boundary conditions ($E_\theta = E_\phi = 0$ at $r = a$ where a is the radius of the cavity in meters), $\hat{J}_p(ka) = 0$. The denumerably infinite set of zeros of $\hat{J}_p(u) = 0$ will be ordered as u_{pq} with $k = (u_{pq}/a)$. Thus, the TE_r mode function becomes

$$(F_r)_{mpq} = F_0 \hat{J}_p\left(u_{pq}\frac{r}{a}\right) P_p^m(\cos\theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}. \tag{3}$$

The resonant frequencies of the cavity may be found by setting $k=2\pi\sqrt{\mu\epsilon}\, f_r$ and solving $k=u_{pq}/a$ for f_r to yield

$$(f_r)_{mpq}^{\mathrm{TE}} = \frac{u_{pq}}{2\pi a\sqrt{\mu\epsilon}}\,. \tag{4}$$

It is apparent that there are numerous degeneracies among the modes since f_r is independent of m. The three lowest-order TE modes are defined as follows.

$$\begin{aligned}(F_r)_{011} &= \hat{J}_1\left(4.493\,\frac{r}{a}\right)\cos\theta \\ (F_r)_{111}^{\mathrm{even}} &= \hat{J}_1\left(4.493\,\frac{r}{a}\right)\sin\theta\cos\phi \\ (F_r)_{111}^{\mathrm{odd}} &= \hat{J}_1\left(4.493\,\frac{r}{a}\right)\sin\theta\sin\phi.\end{aligned} \tag{5}$$

Even and odd superscripts denote the choice of $\cos m\phi$ or $\sin m\phi$, respectively. The three modes have the same mode patterns except for a 90-degree space rotation from one another.

In the following analysis, it is assumed that the cavity is operating in the TE_{011} mode and is filled with a semiconducting material. The TE mode is used rather than a TM mode in order to avoid a singularity at the origin in the cavity analysis. It is also assumed that an impulse of charge is created at the center of the cavity at $t=0$. This could be approximated in practice by placing a small amount of liquid radioactive material between the two surfaces of a cleaved crystal, or by drilling a small hole into which the source would be inserted.

Immediately after creation of the charge impulse through ionization, the charge carriers begin to diffuse and recombine. The diffusion-recombination equation [10] for $t>0$ in this problem is

$$\frac{\partial n}{\partial t} = D\nabla^2 n - \frac{(n-n_1)}{\tau_n} \tag{6}$$

where D is the diffusion constant, τ_n is the carrier lifetime, n is the carrier density, and n_1 is the equilibrium carrier density. If the cavity dimensions are several times larger than the diffusion length, the solution of (6) is readily shown to be

$$n = n_1 + \frac{N_2}{(4\pi Dt)^{3/2}}\, e^{-t/\tau_n} e^{-r^2/4Dt} \tag{7}$$

where N_2 is the number of ionizations, and r is the radial distance from the origin at the center of the cavity.

Cavity Analysis

The shape of the pulse created by the ionization may be obtained through an analysis of the cavity. An exact solution of Maxwell's equations in transient form cannot be accomplished because of the complex form of (7). Also, the recombination effect takes place in a time which is large as compared to a microwave period. This allows a semisteady-state approach, in that $\boldsymbol{E}$ will be assumed to vary only as $e^{j\omega t}$. Since the number of carriers created is generally too small to significantly change the real part of the dielectric constant, it is also assumed that the electric field is undistorted by the ionization. The power dissipated after ionization is given by (8).

$$P_2 = \frac{1}{2}\int_V \sigma_2 E^2 dV = \frac{q^2\tau}{2m^*}\int_V nE^2 dV. \tag{8}$$

In (8), σ_2 is the conductivity after ionization, E is the electric field intensity, q is the charge, τ is the carrier relaxation time, and m^* is the effective mass of the carriers. It has been assumed that τ is unchanged by the plasma-like conditions near the point of ionization. Substitution of (7) into (8) yields

$$P_2 = \frac{q^2\tau}{2m^*}\int_V n_1 E^2 dV + \frac{N_2 q^2 \tau e^{-t/\tau_n}}{(4\pi Dt)^{3/2} 2m^*}\int_V e^{-r^2/4Dt} E^2 dV. \tag{9}$$

The first integral represents the power dissipated with no ionization present and can be rewritten as

$$P_1 = \frac{\sigma_1}{2\epsilon}\int_V \epsilon E^2 dV = \frac{\sigma_1 U}{\epsilon} \tag{10}$$

where ϵ is the permittivity, and U is the maximum energy stored in the cavity. The normalized change in power is then

$$\frac{\Delta P}{P_1} = \frac{\epsilon N_2 e^{-t/\tau_n}}{2n_1 U (4\pi Dt)^{3/2}}\int_V \epsilon^{-r^2/4Dt} E^2 dV. \tag{11}$$

The field for a spherical cavity in the TE_{011} mode has only one component, given by [11]

$$E_\phi = \frac{E_0 \sin\theta}{kr}\left[\cos(kr) - \frac{\sin(kr)}{kr}\right]. \tag{12}$$

In (12), E_0 is a constant, $k=2\pi/\lambda$, and λ is the wavelength. The maximum energy stored for this mode is

$$U = \frac{2\pi\epsilon E_0^2}{3k^3} K, \tag{13}$$

where

$$K = \left[ka - \frac{1+k^2a^2}{ka}\sin^2(ka)\right], \tag{14}$$

and "a" is the radius of the cavity. Since the semiconductor crystal has been assumed several times larger than the diffusion volume, and since most of the recombination takes place in one diffusion volume, little loss in generality is suffered if the integral is taken over an infinite radius. This is indicated in (15), where the angular

$$\frac{\Delta P}{P_1} = \frac{2N_2\epsilon^{-t/\tau_n}}{n_1K(4\pi Dt)^{3/2}}\int_0^{\infty} e^{-(kr)^2/4k^2Dt} \cdot \left[\frac{\sin(kr)}{kr} - \cos(kr)\right]^2 d(kr) \qquad (15)$$

integrations have already been performed. Evaluating the integral in (15), one finds

$$\frac{\Delta P}{P_1} = \frac{N_2k}{n_1K}\frac{e^{-t/\tau_n}}{4\pi Dt}\left[\frac{1}{2} - \frac{1}{4k^2Dt} + \left(\frac{1}{2} + \frac{1}{4k^2Dt}\right)e^{-4k^2Dt}\right]. \qquad (16)$$

The singularity at $t=0$ should be avoided. It is unrealistic due to the impulse conditions assumed at the origin and $t=0$. Equation (16) should, however, give good results for $t>0$. Figure 1 shows the further normalized results for high purity CdS. The literature values [12] used for the various parameters were $\tau_n = 10^{-3}$ sec, $D=13$ cm^2/volt-second, and $\epsilon = 5.62\epsilon_0$. Computations were based on the assumption of a frequency of 46.5 Gc/s.

An interpretation of Fig. 1 can be made in terms of the physical aspects of the problem. The initial decrease in attenuation is due to the "spreading" of electrons which results in the deterioration of the plasma-like conditions at the origin for $t=0^+$. The peak in attenuation can be attributed to the fact that the maximum field intensity does not occur at the origin for the particular mode chosen. As charge carriers diffuse outward from the origin, a "front" or a maximum in the charge density passes through the point of maximum field intensity. Finally, recombination and passage of the "front" cause the decrease of attenuation toward its initial value. The unique "peaking" effect obtained with this mode raises the possibility of basic measurements of the properties of solid-state plasmas. Also, the height of the pulse leads to the expectation that such a method might be readily utilized for radiation detection.

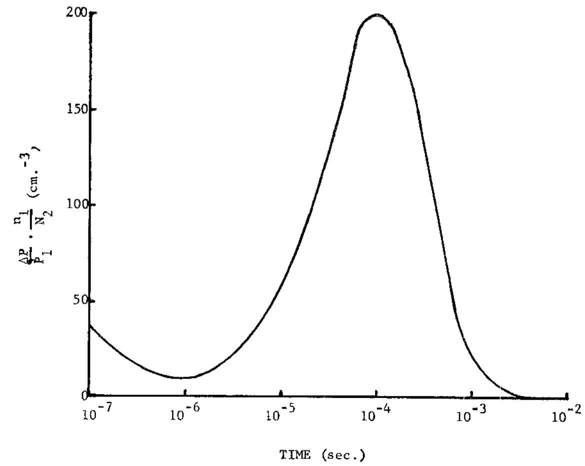

Fig. 1. The variation in normalized power as a function of time for a CdS filled spherical cavity operating in the TE_{011} mode at 46.5 Gc/s.

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